MATHEMATICS: PAPER II

Time: 3 hours   150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1.   This question paper consists of 10 pages, a Diagram Booklet of 3 (i to iii) pages and an Information Sheet of 2 (i to ii) pages. Please check that your question paper is complete.

2.   Please detach the Diagram Booklet from the middle of your question paper. Write your examination number in the spaces provided on your Answer Book and the Diagram Booklet.

3.   Any changes made to a diagram must be shown on the Diagram Booklet and not on the question paper. Please hand in the Diagram Booklet with your Answer Book.

4.   Answer ALL the questions.

5.   Please note that diagrams are not necessarily drawn to scale.

6.   All necessary working details must be shown.

7.   Approved non-programmable and non-graphical calculators may be used, unless otherwise stated.

8.   Ensure that your calculator is in **DEGREE** mode.

9.   Round off your answer to two decimal digits, unless otherwise stated.

10.   It is in your own interest to write legibly and to present your work neatly.

11.   Please hand in this question paper.

SECTION A

QUESTION 1

In the diagram below, the cumulative frequency curves for the amount of money spent on clothes by separate groups of boys and girls are given.
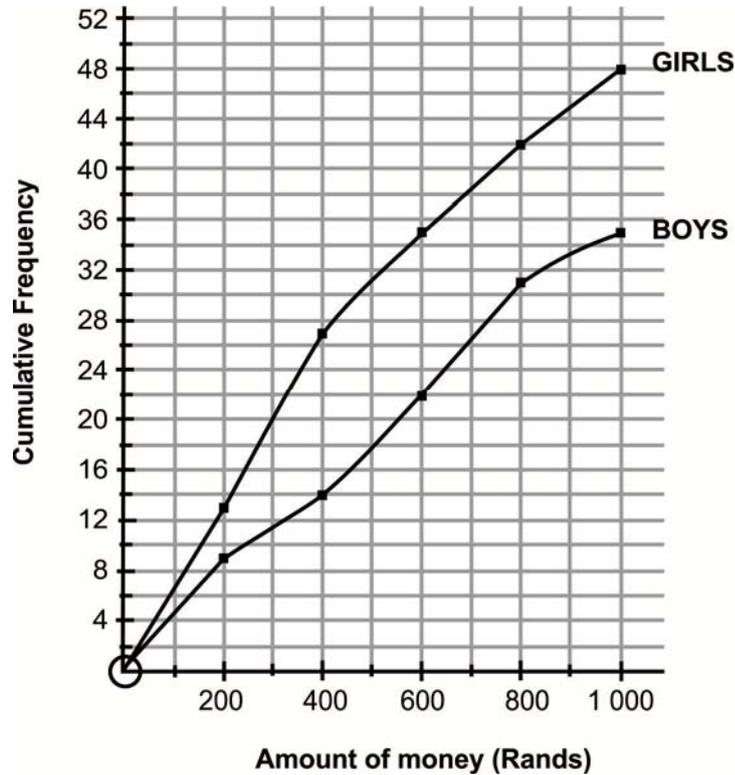
(a)   How many students (boys and girls) were surveyed in total?   (2)

(b)   Determine the median amount of money spent by each of the groups.   (2)

(c)   Estimate the total number of boys and girls who spent R700 or less on buying clothes.   (2)

(d)   What percentage of boys spent less than R200? Give your answer correct to the nearest whole number.   (2)

(e)   Consider the following claim made by a student:

'More girls than boys spend less than R200 on buying clothes'

If the percentage of boys versus the percentage of girls is used to argue for or against the claim, decide (showing calculations) whether the claim is true or false.   (3)

[11]

QUESTION 2

(a)   In the diagram below, $\triangle ABC$ undergoes two transformations to obtain $\triangle A'B'C'$.
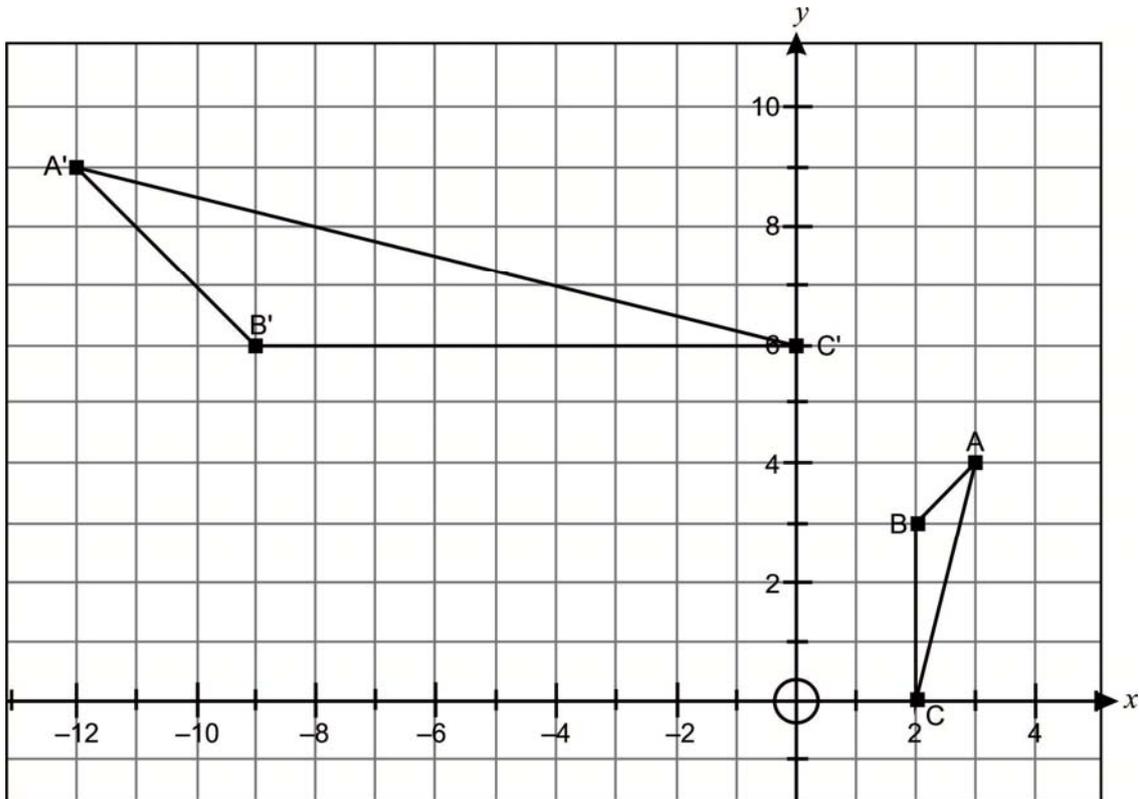
(1)   Write down the two transformations separately in the form of $(x; y) \rightarrow \ldots$ (4)

(2)   Write down the values of

(i) $\dfrac{\text{Area } \triangle A'B'C'}{\text{Area } \triangle ABC}$ (1)

(ii) $\dfrac{\text{Perimeter } \triangle A'B'C'}{\text{Perimeter } \triangle ABC}$ (1)

(3)   Write down the co-ordinates of B", the image of B', if $\triangle A'B'C'$ is rotated about the origin through 180°.   (2)

(4)   Write down the co-ordinates of C", the image of C', if $\triangle A'B'C'$ is reflected about the y-axis.   (2)

(b)   In the diagram, the graph of f(x) = 2 is sketched.
Write down the equation of the resulting graph
if all the points on f(x) are
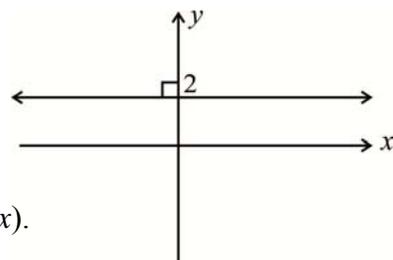
(1)   transformed according to the rule $(x; y) \rightarrow (y; x)$.   (1)

(2)   reflected about the x-axis.   (1)

(3)   translated according to the rule $(x; y) \rightarrow (x - 2; y + 1)$.   (1)

[13]

QUESTION 3

(a)   In the diagram below, ABDC is a parallelogram.

The equation of the line passing through A and B is given by $y = \frac{1}{4}x + 6$.

The line passing through A and C also passes through the origin.
$\hat{BAC} = 59°$, $\hat{BDO} = \beta$ and $\hat{CDO} = \theta$.
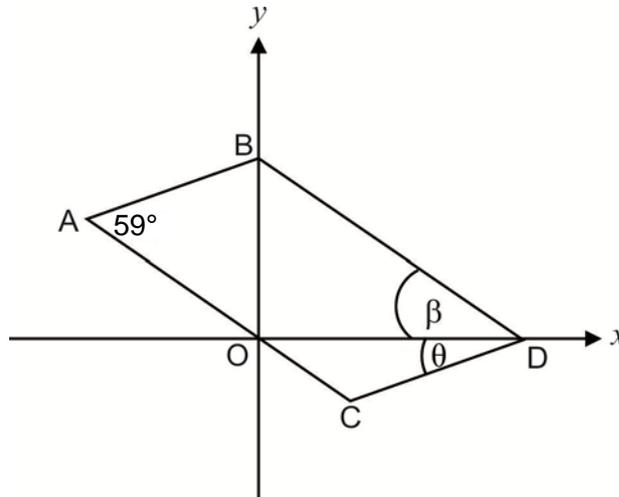
(1)   Determine, **correct to the nearest degree**, the size of:

     (i) θ (4)

     (ii) β (2)

(2)   Determine the equation of AC.   (2)

(3)   Determine the equation of BD.   (2)

(4)   Determine the co-ordinates of D.   (2)

(b)   (1)   Determine the centre and the radius of the circle with equation
     $(x + 2)^2 - 12 = 4y - y^2$.   (4)

     (2)   Determine the equation of the tangent to the circle at the point $P(2;2)$.   (2)

[18]

QUESTION 4

PLEASE ENSURE THAT YOUR CALCULATOR IS IN DEGREE MODE

(a)   ΔPQR is drawn.
$\hat{P} = 40°$, $\hat{Q} = 74°$ and PR = 12 cm.

Determine, correct to the nearest cm², the area of ΔPQR.   (6)

(b)   Show that

$$\frac{\cos(180° - 2\theta)}{1 - \tan^2 \theta} = -\cos^2 \theta$$ (4)

(c)   If $\sin\alpha = 6\cos\alpha$, determine without using a calculator, the value of:

(1) $\tan\alpha$ (1)

(2) $\dfrac{\sin(\alpha - 45°)}{\cos\alpha}$ (4)

[15]

QUESTION 5

(a)   In the diagram below, the graphs of $y = a \sin b\theta + c$, $y = d \cos \theta$ and a third trigonometric graph are sketched for the interval $[0°, 360°]$.
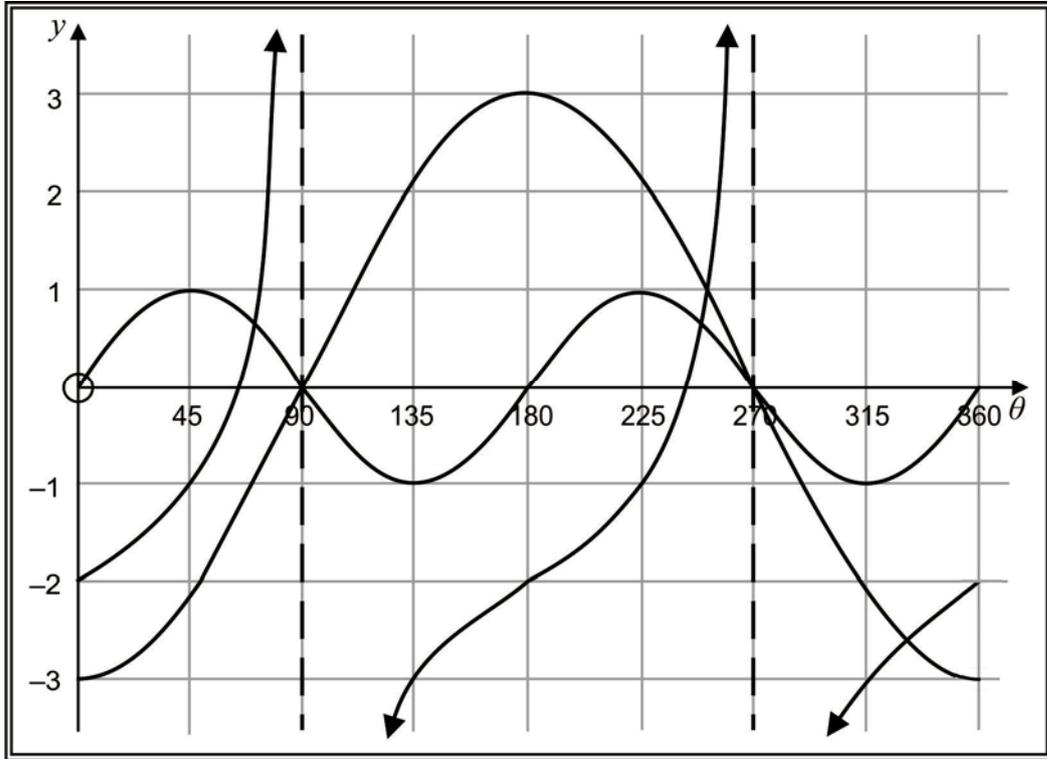
(1)   Write down the values of a, b, c and d.   (4)

(2)   Use the graphs to solve the following for $\theta \in [0°, 360°]$:

(i) $a \sin b\theta + c = d \cos \theta$ (2)

(ii) $a \sin b\theta + c < d \cos \theta$ (2)

(3)   The equation of the third graph sketched is given by $y = \tan p\theta + q$. Determine the values of p and q.   (5)

(b)   In the diagram below, TABCD is a right pyramid with a square base side of 8 metres.
       M is the midpoint of BC and TN is the height of the pyramid.
       Each of the triangular faces makes an angle of θ with the square base.
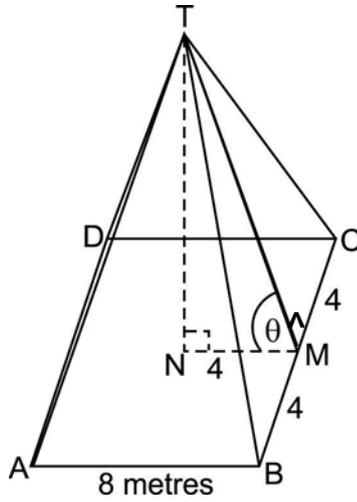
(1)   Express TM in terms of θ.   (2)

(2)   If the total surface area of the pyramid is equal to 256 square metres, determine the value of θ, correct to 1 decimal digit.   (5)

                                                                                    [20]

                                                                        ┌──────────────┐
                                                                        │ **77 marks** │
                                                                        └──────────────┘

SECTION B

QUESTION 6

(a)   In the diagram below, two concentric circles are drawn with centre A on the *y*-axis.
        The smaller circle cuts the *y*-axis at the origin O and the point B .

        The line through B having equation $y = \dfrac{3}{2}x + 6$ cuts the *x*-axis at R.

        The larger circle passes through R.
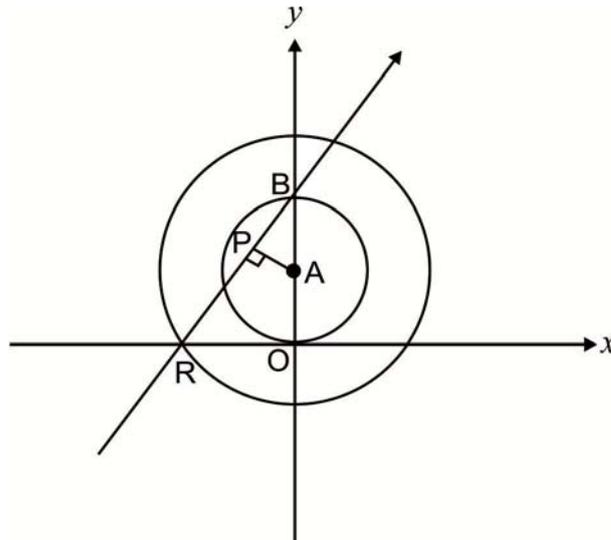
        (1)   Determine the equations of both circles.   (6)

        (2)   Determine the coordinates of P, the point on the line passing through B and
                R, which is closest to point A.   (6)

(b)   Tangents to a circle with centre the origin, touch the circle at E (4 ; 7) and
        F (8 ; 1). The tangents intersect at G.
        Determine the *x* co-ordinate of G. (Hint: Join OE and OF.)
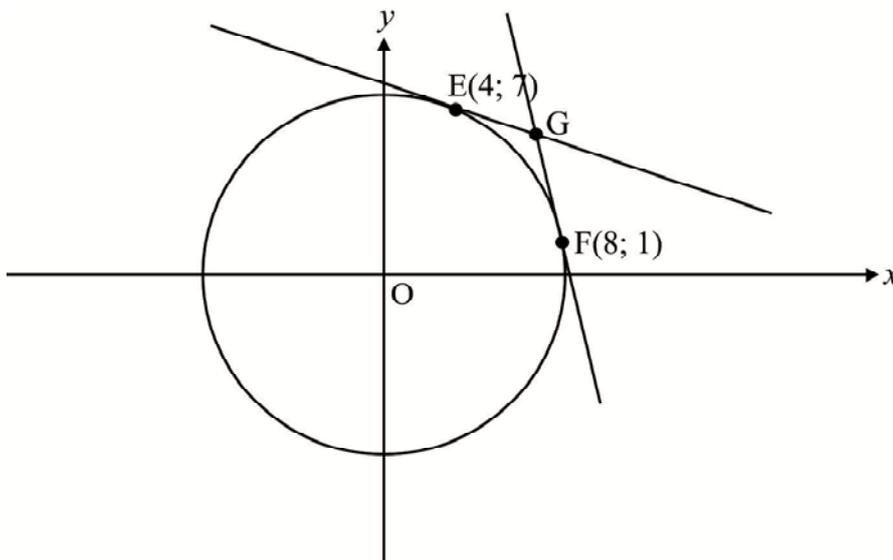
                                                                                            (8)
                                                                                          [20]

QUESTION 7

The weight of luggage belonging to each of 135 passengers on an aeroplane is summarised in the following table:

Weight (kg)	Number of passengers
$0 \leq w < 10$	23
$10 \leq w < 20$	28
$20 \leq w < 30$	31
$30 \leq w < 40$	4
$40 \leq w < 50$	33
$50 \leq w < 60$	16

The diagram below is a box and whisker plot of the raw data summarised in the table.
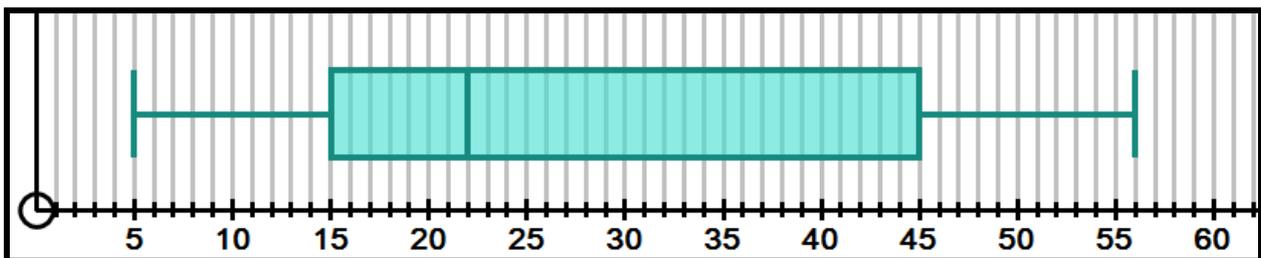
(a)   Estimate the number of passengers with luggage in each of the following weight classes.

     (1) $15 \leq w < 22$ (2)

     (2) $22 \leq w < 45$ (1)

(b)   Calculate an estimate for the mean and the standard deviation of the luggage weights. Round your answer to 1 decimal digit.   (6)

(c)   The scales used for weighing the luggage had been correctly calibrated. Estimate the mean and standard deviation of the actual luggage weights if the weighing had taken place before the calibration and the recorded weight was 6 % greater than the actual weight.   (4)

[13]

QUESTION 8

A CALCULATOR IS NOT TO BE USED IN THIS QUESTION.

(a)   The point $T\left(1-\sqrt{2};\,1+\sqrt{2}\right)$ is rotated through an angle of $45°$ in an anticlockwise direction. Determine the coordinates of the image point, T'. Leave your answer in simplest surd form.   (6)

(b)   A point $K\left(3a;\,5b\right)$ is rotated clockwise by $90°$ to a point $K'\left(b-8;\,6b\right)$.
Determine the values of a and b.   (5)

[11]

QUESTION 9

(a)   Solve the trigonometric equation
$\cos(2\theta) = \cos(\theta + 60°)$ where $\theta \in \left(-360°,\, 0°\right)$.   (6)

(b)   Simplify without using a calculator. Show all working clearly.

$$\frac{\tan 156°.\cos 114°}{\cos 744°} - \frac{1}{\sin^2\left(-66°\right)}$$ (8)

[14]

QUESTION 10

In the diagram below, ABCD is a hillside with an inclination of $30°$.
E, J, K, C and D are points in the same vertical plane.
A, B, J and K and E are in the same horizontal plane.
ABCD and ABKJ are rectangles with D, P and C vertically above J, E and K respectively.
BP is a straight path on the hillside which makes an angle of $60°$ with BC.
PT is a vertical tower.
AB = 2 000 metres, BP = 1 000 metres and PT = 50 metres.
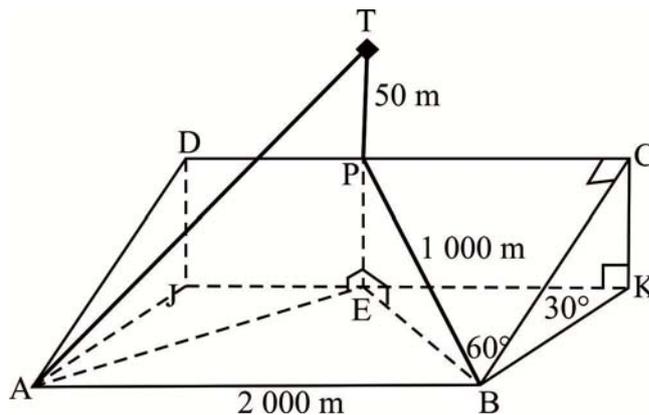
(a)   Determine the lengths of BC, CK and BK. (Leave answers in surd form, if necessary.)   (6)

(b)   Determine the length of PC. (Leave answer in surd form.)   (2)

(c)   Determine AT, correct to the nearest metre.   (7)

[15]

> **73 marks**

Total: 150 marks